

# Strategic Decision Making In Yacht Match Racing: Stochastic Game Approach

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## Résumé :

A sailing yacht match race is a race between two yachts with rather similar performances competing against each other with a given set of racing rules, where the tactical decisions from the skippers are decisive and complex as many parameters come into play such as the wind fluctuations and the opponent's actions, both items being partly expected and partly stochastic. Strategy plays a major role and sailors continuously have to make their decisions according to the wind variations and the opponent's position and actions. This paper shows how the strategic decision-making in sailing yacht match racing corresponds to the settings of stochastic games, considering the wind fluctuations and the opponent's actions. Even if the model is simplified with strong assumptions with respect to the real game of match racing, the new proposed formalism with a two-player stochastic game approach permits some insights into strategic decision making and will be helpful for sailors training after some future developments to get closer to the real race.

**Mots-clés :** Stochastic Game, Zero-sum Game, Sailing Match Race

## 1 Introduction

To win the race, sailors must not only be skilled to sail the yacht fast in the prevailing sailing conditions, but also be able to make efficient strategic decisions. Spatial inhomogeneities of the wind may be present on the race course, and local and temporary wind fluctuations usually occur during a race which sailors have to use to optimize their route to win the race. The strategy is an important part of the race and is often predominant (Philpott *et al.*, 2004) particularly when the yachts have similar performances and sailors are good enough to sail their yacht at the optimal performance. Thus, to win a race, the naive strategy is to look for the route minimizing the time to complete the course, which is the matter of weather routing. However, when racing against an opponent in an uncertain wind, the opponent's behavior should be considered and a better strategy is to try to maximize the probability to reach the finish line before the opponent (Tagliaferri *et al.*, 2014).

This paper presents theoretically, a stochastic game model of strategic decision-making in a match racing upwind leg based on the dynamic and uncertain wind and the opponent actions. The model proposed in the present work considers two non-cooperative players. Each player corresponds to a yacht and wants to win. A new state results from the current state, the selected yachts' actions and the dynamic and uncertain wind. To solve this game is finding the winning strategy. To our knowledge there is no model for match racing based on game theory taking into account the opponent's behavior and the wind fluctuations at the same time.

## 2 Match Racing and Simplified Problem Addressed

A sailing yacht match race is a duel between two similar yachts under a given set of rules, mostly on an upwind-downwind course, where the aim is not necessarily to complete the course in the shortest possible time, but to reach the finish line before the opponent. The winner is the one who reaches the upwind mark first.

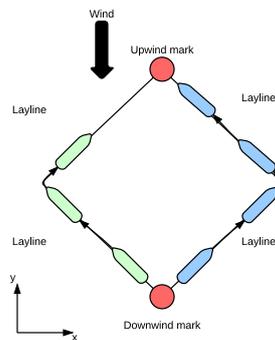


FIGURE 1 – Schematic representation of the course. The y axis is oriented in the direction of the upwind mark (initial wind direction).

For brevity's sake, only an upwind leg is considered and the race course may be represented as shown on Figure 1. We focus on the strategic decision-making and do not address the yacht performance nor the crew ability to trim and drive the yacht. Hence, we will consider identical yachts always sailing upwind at their best VMG (Velocity Made Good) which is assumed to be known. Therefore, each yacht has two possible actions : to tack or not to tack. Other strong assumptions made are that we have not considered yet the racing rules nor the effect on a yacht's performance of the proximity of the opponent. Thus, only situations where both yachts are not too close to each other are realistically modelled.

### 2.1 Weather routing

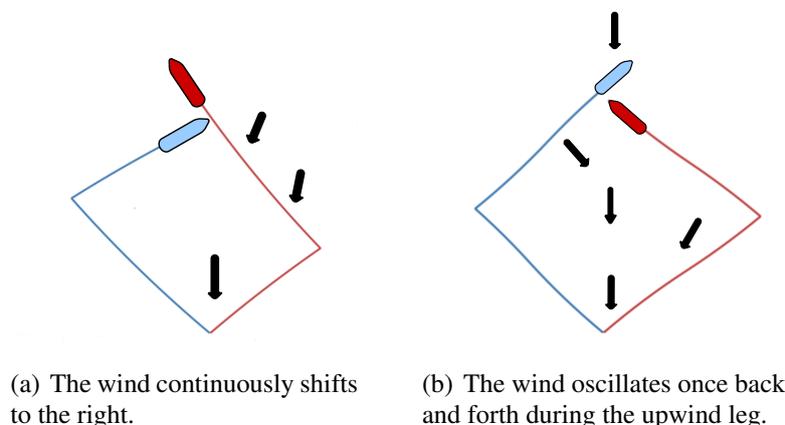


FIGURE 2 – Wind fluctuations in two basic situations.

Weather routing is finding the fastest path by considering yacht performance and the wind and current variations. In an upwind leg, a yacht cannot sail straight into the wind and has to zig-zag towards the upwind mark (Figure 1).

In real situations, the wind direction and speed vary in space and time, and the optimal route includes several tacks to take advantage of the wind shifts, provided the gain exceeds the loss due to tacking. Two typical simple situations are described to illustrate some of the basics in weather routing (Figure 2) :

1. The wind is continuously shifting to one side -say right- during the upwind leg ; the optimal route includes one tack and starts going right, as the opposite strategy -going left- leads to a longer path (Figure 2(a)).
2. The wind oscillates with firstly a shift to the right, then shifting back to its initial direction, then shifting to the left and finally shifting back to its initial direction, during the upwind leg ; the optimal route includes one tack and starts going left while the wind is shifted to the right, then tacking when the wind is shifted to the left, to always maximize the projected velocity towards the upwind mark (Figure 2(b)).

## 2.2 Adversarial situations

When racing against an opponent in an uncertain wind, the opponent's behavior should be considered and the strategy should include some risk management. Yacht's relative locations and behaviors are key elements during the race.

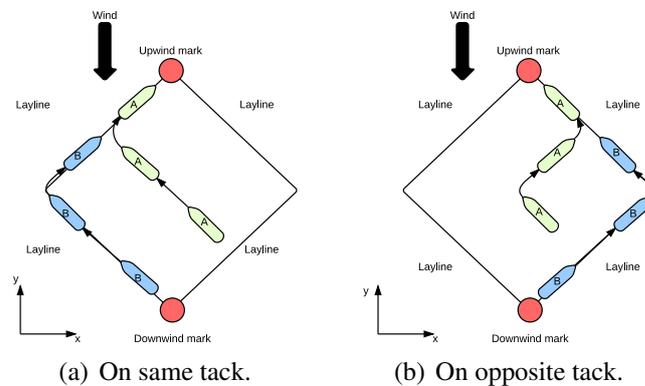


FIGURE 3 – Adversarial Situations. Yacht A behavior.

Let us consider for example yachts A and B where A is leading and B is trailing (Figure 2.2). We consider yacht A behavior. To achieve its goal and considering an upwind leg, yacht A reacts to yacht B behavior.

1. B on the same tack ; A continues until the layline and tacks once to limit the tacking cost (Figure 3(a)).
2. B on the opposite tack ; A tacks to minimize risk. In this case, A achieves one more tack but minimises the risk by a loose covering (Figure 3(b)).

Therefore, the strategy is to choose a route which maximizes the probability to reach the upwind mark before the opponent, as shown in (Tagliaferri *et al.*, 2014).

### 3 Match Racing as a Stochastic Game

We tackle our decision making problem using a stochastic game model. Formally, a stochastic Game (Shapley, 1953) is a tuple  $\langle Ag, \{A_i : 1, \dots, n\}, \{R_i : 1, \dots, n\}, S, T \rangle$  :

1.  $Ag$  is a finite set of players indexed  $1, \dots, n$  ;
2.  $A_i$  is a finite set of actions available to player  $i$  and  $\langle a_1, \dots, a_n \rangle \in \times_i A_i$  denotes a vector of the joint action of the players ;
3.  $R_i : S \times \vec{A} \rightarrow \{-1, 0, 1\}$  is a reward function for the  $i$ th player ;
4.  $S$  is a finite set of states ;
5.  $T : S \times \vec{A} \times S \rightarrow [0, 1]$  is a transition function. It indicates the probability of moving from a state  $s \in S$  to a state  $s' \in S$  by executing the joint action  $\vec{a}$ .

#### 3.1 Players, actions and states

Actors in a game are the players whose intents are to either maximize gains or minimize losses. To model the match racing as a game, we assume that a player represents the skipper and his yacht. Thus, sailing match racing is a game with two players ( $n = 2$ ).  $b_i$  denotes a player where  $i = \{1, 2\}$ . During the race, at a given speed, each player  $b_i$  has only two possible actions : **to tack** or **not to tack**. Executing a tack action forces the player to turn through the wind and slow down. We assume a known speed loss for tacking.  $A_i$  denotes the set of actions for each player  $b_i$ ,  $(A_i)_{i \in \{1,2\}} = \{\text{to tack, not to tack}\}$ .  $\vec{a} = (a_1, a_2)$  denotes the joint action of players  $b_1$  and  $b_2$  ( $\vec{a} \in \{\vec{a}_{(t,t)}=(\text{tack, tack}), \vec{a}_{(t,n)}=(\text{tack, not to tack}), \vec{a}_{(n,t)}=(\text{not to tack, tack}), \vec{a}_{(n,n)}=(\text{not to tack, not to tack})\}$ ).

Our game consists in a finite set of states  $S = \{s^0, s^1, \dots, s^h\}$ . Each state corresponds to the yachts situations and the wind direction and speed. Formally, a state  $s$  is a tuple  $s = \langle Y_1, Y_2, W \rangle$  :

1. Yacht situation ( $Y_i$ ) corresponds to the yacht ( $b_i$ ) internal state. Yacht situation describes location, direction and speed of a given yacht. Formally,  $Y_{b_i} = \langle pos, \vec{v} \rangle$ , where ; (1)  $pos = \langle x, y \rangle$  denotes the current location in the race and (2)  $\vec{v} = \langle \theta, v \rangle$  denotes the current yacht direction and speed.
2. Wind direction and speed,  $W \in \{-45^\circ, -40^\circ, \dots, 0^\circ, \dots, 45^\circ\} \times \mathbb{R}^+$ , where  $0^\circ$  represents the direction to the upwind mark and the other states represent shifts of  $\pm 5^\circ$  from that direction (Tagliaferri *et al.*, 2014).

#### 3.2 Transition

The transition function captures the stochastic nature of the yacht's motion. Our game transits from one state to another according to a probability distribution. The state transition probability is a function of both the players' actions, and the current state. Formally, the transition function  $T(s^k, \vec{a}, s^{k+1}) = Pr(s^{k+1} | s^k, \vec{a})$  is the probability that the system transits to state  $s^{k+1}$  when the joint action  $\vec{a}$  is taken at state  $s^k$ . The transition function is based on the predicted yacht positions. The transition function  $T(s^k, \vec{a}, s^{k+1}) = Pr(shift)$  corresponds to the probability that the wind shifts. Therefore, we focus on changes in wind direction that affect the race. We adopted the wind transition defined in (Tagliaferri *et al.*, 2014).

### 3.3 Reward

During the race the challenge for yacht  $b_1$  is to stay ahead the opponent  $b_2$ . We define this advantage as the estimated time to reach the upwind mark from the current location. During the race, when a yacht  $b_1$  makes a decision to tack or not to tack, it evaluates the remaining time to reach the upwind mark. Formally, we define, in the same way for the two yachts, a function denoted  $\Gamma_1(s)$  estimating the time to reach the upwind mark ( $UM$ ). This function depends on the current yacht location relative to the upwind mark ( $dist(pos^{UM}, Y_i.pos)$ ), where; (1)  $Y_i$  denotes the current state ( $s$ ) of yacht  $b_i$  and (2)  $pos^{UM} = (x^{UM}, y^{UM})$  denotes the location of the upwind mark. At each state  $s$ , according to  $\Gamma_i(s)$ , there are three cases : (1) yacht  $b_1$  takes less time than the opponent  $b_2$  to reach the upwind mark. Yacht  $b_1$  is the leader, (2) yacht  $b_1$  takes more time than the opponent  $b_2$  to reach the upwind mark. Yacht  $b_1$  is the follower and (3) both players are to a tie. We define, in the same way for both yachts, the pay-off function  $p_i(s)$  for each state :

$$p_1(s) = \begin{cases} 1 \Leftrightarrow \Gamma_1(s) < \Gamma_2(s) & \text{if } b_1 \text{ is the leader} \\ -1 \Leftrightarrow \Gamma_1(s) > \Gamma_2(s) & \text{if } b_1 \text{ is the follower} \\ 0 \Leftrightarrow \Gamma_1(s) = \Gamma_2(s) & \text{otherwise.} \end{cases}$$

To win the race, a yacht  $b_i$  must reach the upwind mark first. We set the reward for the goal state to a positive or negative value. The reward function  $R_i$  is defined in the same way for both yachts :

$$R_i = \begin{cases} 1 & \text{if } b_i \text{ wins} \\ -1 & \text{if } b_i \text{ loses} \\ 0 & \text{otherwise.} \end{cases}$$

### 3.4 Decision model

At each step  $k$ , we formulate the state  $s^k$  as a two-dimensional matrix game  $G^k$  (at each state the player is faced to a game). A matrix game  $G^k$  indicates pay-offs for  $b_1$  and  $b_2$ . We allow one player  $b_1$  to choose the rows and the other  $b_2$  the columns. The entries in the matrix are the pay-offs to the row player. An important property for this game is  $p_1(s) = -p_2(s)$  which means that a pay-off for one player is a cost for the other. At each state  $s^k$ , each player  $b_i$  has a finite set of actions at his disposal to choose from  $(A_i)_{i \in \{1,2\}} = \{\text{to tack, not to tack}\}$ . The resulting bi-matrix therefore contains the matrix game for both players.

$$G^k = \begin{matrix} & \begin{matrix} \text{to tack} & \text{not to tack} \end{matrix} \\ \begin{matrix} \text{to tack} \\ \text{not to tack} \end{matrix} & \left( \begin{array}{cc} p_1(s^k) & , & p_1(s^k) \\ p_1(s^k) & , & p_1(s^k) \end{array} \right) \end{matrix}$$

Different states can lead to the same game  $G$ . Let this set be  $S_G$ . When acting, two players transit from one state  $s^k$  to another state  $s^{k+1}$  leading to a transition from one game to another. Transitions from game  $G^k$  to another game  $G^{k+1}$  depend on the outcome of  $G^k$  and the joint action given by the matrix game  $G^k$ . The probability of transiting from game  $G^k$  when acting with a joint action  $\vec{a}$  is the probability to transit from one state of  $S_{G^k}$  to a state of  $S_{G^{k+1}}$ . Players simultaneously choose a row and a column of the matrix game causing player  $b_1$  to win the pay-off  $p_1(s^k)$  from player  $b_2$  who loses the same amount. Then, the game moves to another state,

with a probability that depends on the selected joint action and the current state. Formally, for each state  $s^k$ ;  $Pr(G^k, \vec{a}, G^{k+1}) = \sum_{s^{k+1} \in S_{G^{k+1}}} T(s^k, \vec{a}, s^{k+1})$ .

For each matrix game, the game value is the unique solution of  $G^k$  with game values given as  $V(k)$ . However, for a long-term race, the strategy should maximize the gain expectation. To this end, we propose a value function for each game  $G^k$  at each state  $s^k$ , as follows ;

$$V(k) = \max_{\vec{a}} \begin{cases} \sum_{s^k, G^{k+1}} Pr(G^k, \vec{a}, G^{k+1}) V(k+1) & k < h \\ R_i & k = h. \end{cases}$$

$Pr(G^k, \vec{a}, G^{k+1})$  denotes the probability of transiting from game  $G^k$  when acting with a joint action  $\vec{a}$ . Solving this problem we can use a value iteration algorithm (Littman, 1994). There is a mixed strategy for player  $b_1$  such that  $b_1$ 's average gain is at least  $V$  no matter what  $b_2$  does and there is a mixed strategy for player  $b_2$  such that  $b_2$ 's average loss is at most  $V$  no matter what  $b_1$  does. Also, if  $V = 0$ , the game is neutral. If  $V > 0$  the game is said to be favorable to player  $b_1$ , otherwise if  $V < 0$  the game favors player  $b_2$ . The row player is trying to match the column player and the column player is trying to guess the opposite of the row player. The value of the game may be calculated as either the minimum of what the row player can achieve knowing the strategy of the column player or the maximum of what the column player can hold the row player to, knowing the strategy of the row player.

### 3.5 Sailing yacht match racing example

We consider two yachts  $b_1$  and  $b_2$ . Both yachts have exactly the same distance to reach the upwind mark.  $b_1$  is sailing on starboard tack and is located to the left of  $b_2$  sailing on port tack. The yachts speed is  $v = 13kt$  and the speed loss when tacking is  $3kt$ . The wind speed is constant. We focus on a finite horizon  $h$  ( $h = 10$ ,  $\delta t = 30s$ ), where the wind shifts to the left (*shift* =  $-5^\circ$ ) twice, at times  $k = 4$  and  $k = 8$ . We focus on  $b_1$  behavior. We assume the following initial state :

$$s^0 = \begin{cases} Y_{b_1} = \langle 15, 130, -45^\circ, 13 \rangle \\ Y_{b_2} = \langle 25, 130, +45^\circ, 13 \rangle \\ W = \langle 0^\circ, windspeed \rangle. \end{cases}$$

At step  $k = 1$ , the wind direction remains the same ( $W^1 = 0^\circ$ ). There are 4 possible situations and 1 possible matrix game ( $G_1^1$ ) according to joint actions  $\vec{a}_{(t,t)}$ ,  $\vec{a}_{(t,n)}$ ,  $\vec{a}_{(n,t)}$  and  $\vec{a}_{(n,n)}$  :

$$G_1^1 = \begin{matrix} & \text{to tack} & \text{not to tack} \\ \text{to tack} & \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right) \\ \text{not to tack} & & \end{matrix}$$

1. if both yachts  $b_1$  and  $b_2$  tack ( $\vec{a}_{(t,t)}$ ), there is no leader at step  $k = 1$  ;
2. if yacht  $b_1$  tacks and yacht  $b_2$  doesn't tack ( $\vec{a}_{(t,n)}$ ),  $b_2$  is the leader at step  $k = 1$  ;
3. if yacht  $b_1$  doesn't tack and yacht  $b_2$  tacks ( $\vec{a}_{(n,t)}$ ),  $b_1$  is the leader at step  $k = 1$  ;
4. if yachts  $b_1$  and  $b_2$  do not tack ( $\vec{a}_{(n,n)}$ ), there is no leader at step  $k = 1$  ;

For each time step  $k$ , we repeat the same operations in order to compute states in the next step  $k + 1$  according to joint actions and current state, considering possible wind shifts. This allows the matrices games to be determined. This process provides a decision tree. A winning strategy for yacht  $b_1$  consists in finding a path in the global decision tree from  $s^0$  to  $s^h$ . Formally, computing a winning strategy consists in computing recursively  $V(k)$  up to the final state. The computed value assigns to each state the maximum expected value expressing the probability to win and considers for this state the action maximizing this probability.

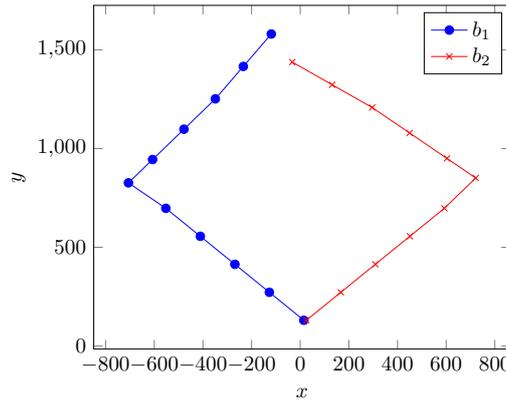


FIGURE 4 – One winning strategy for yacht  $b_1$ .

During the first part of the race, at steps from  $k = 0$  up to  $k = 3$  with the initial wind ( $W = 0^\circ$ ), both yachts  $b_1$  and  $b_2$  do not tack. Thus, there is no leader in time step  $k = 4$ . In step  $k = 4$  the wind shifts to the left ( $shift = -5^\circ$ ). Again, both yachts  $b_1$  and  $b_2$  do not tack. Yacht  $b_2$  becomes the leader as his course is closer to the direction of the upwind mark. At time step  $k = 5$ , the wind is to the left ( $W = -5^\circ$ ), both yachts  $b_1$  and  $b_2$  tack. Yacht  $b_2$  remains the leader at step  $k = 6$  but the gain towards the mark is higher for  $b_1$ . From this step, yacht  $b_1$  takes the lead. Indeed, the wind direction is more favorable to  $b_1$  than to  $b_2$ . This is reflected into the following matrices games. During the first part of the race, at steps from  $k = 0$  up to  $k = 4$ , it is the same matrix game, such as :

$$G_1^1 = G_1^2 = G_1^3 = G_1^4 = \begin{matrix} & \text{to tack} & \text{not to tack} \\ \text{to tack} & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ \text{not to tack} & \end{matrix}$$

According to these matrices games and Figure 4, both yachts  $b_1$  and  $b_2$  have decided not tack. At time step  $k = 5$  and time step  $k = 6$ , the wind is to the left ( $W = -5^\circ$ ), For any considered joint action, yacht  $b_2$  is the leader. This is reflected into the following matrices games, such as :

$$G_1^5 = G_1^6 = \begin{matrix} & \text{to tack} & \text{not to tack} \\ \text{to tack} & \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \\ \text{not to tack} & \end{matrix}$$

At step  $k = 7$ , as long as yacht  $b_1$  does not tack, he is the leader. This is reflected into the following matrices games, such as :

$$G_1^7 = \begin{array}{cc} & \begin{array}{cc} \text{to tack} & \text{not to tack} \end{array} \\ \begin{array}{c} \text{to tack} \\ \text{not to tack} \end{array} & \left( \begin{array}{cc} 0 & , & -1 \\ 1 & , & 1 \end{array} \right) \end{array}$$

At steps from  $k = 8$  up to  $k = 10$ , for any considered joint action, yacht  $b_1$  remains the leader. This is reflected into the following matrices games, such as :

$$G_1^8 = G_1^9 = G_1^{10} = \begin{array}{cc} & \begin{array}{cc} \text{to tack} & \text{not to tack} \end{array} \\ \begin{array}{c} \text{to tack} \\ \text{not to tack} \end{array} & \left( \begin{array}{cc} 1 & , & 1 \\ 1 & , & 1 \end{array} \right) \end{array}$$

The outcomes from the model on this simple scenario are consistent with the well-known strategy basics that if the wind shifts to one side during an upwind leg, the yacht positioned to that side takes an advantage, as seen in Section 2.

#### 4 Conclusions and Future Work

This work introduces a theoretical model of stochastic game between two non-cooperative players is proposed and coupled to the stochastic wind MDP model developed in (Tagliaferri *et al.*, 2014), to account for both wind fluctuations and yachts' reactions to their opponent position and actions. The method is applied to a simple race scenario and shows results in agreement with the basic sailing strategy knowledge.

Strong assumptions to simplify the problem have been made, and further developments are needed to extend the practical applicability to the real match racing game. The main considered improvements are to enhance the pay-off and transition functions according to : (1) Modification of each yacht performance due to the vicinity of the other yacht altering the wind, (2) Modeling the racing rules and yachts' reactions to obey the rules or penalize them otherwise, . . .

Nevertheless, in this highly complex game, this work proposes a new formalism to model a stochastic game to investigate the decision making with respect to the opponent's position and actions which should help integrate this issue in more complete models of match racing strategy. Moreover, it is believed that the simplified model proposed in this work can already give some insight into match racing strategy when yachts are far enough from each other in order that racing rules and wind shadow do not come into play.

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